OCR
RECOGNISING ACHIEVEMENT

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Advanced Subsidiary General Certificate of Education**
**Advanced General Certificate of Education**

# MATHEMATICS 2639

Mechanics 3

Friday **23 JANUARY 2004** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

**TIME** 1 hour 20 minutes

## INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use $9.8\,\mathrm{m\,s^{-2}}$.
- You are permitted to use a graphic calculator in this paper.

## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

**This question paper consists of 4 printed pages.**

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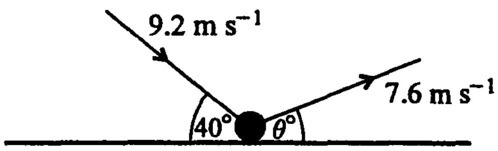
1




A smooth sphere of mass 0.5 kg bounces on a fixed horizontal surface. Just before the bounce the sphere has velocity 9.2 m s$^{-1}$ at 40° to the horizontal, and immediately after the bounce it has velocity 7.6 m s$^{-1}$ at $\theta°$ to the horizontal (see diagram).

(i) Find $\theta$. [2]

(ii) State the direction of the impulse on the sphere during the bounce, and find its magnitude. [3]

2 A particle $P$ of mass $m$ moves along a straight line, and $O$ is a fixed point on the line. When $P$ is at a distance $x$ from $O$, the only force on $P$ has magnitude $kx^3$ and acts towards $O$. The particle is at rest when $x = a$.

(i) Find the speed of $P$ in terms of $k$, $m$, $x$ and $a$. [5]

(ii) Hence find the speed of $P$ when it reaches $O$. [2]

3 A light elastic string has natural length 3.2 m and modulus of elasticity 792 N. One end is attached to a fixed point $O$ and the other end is attached to a particle of mass 5 kg. The particle is released from rest in a position 4.0 m vertically below $O$. Air resistance may be neglected.

(i) Find the acceleration of the particle immediately after it is released. [4]

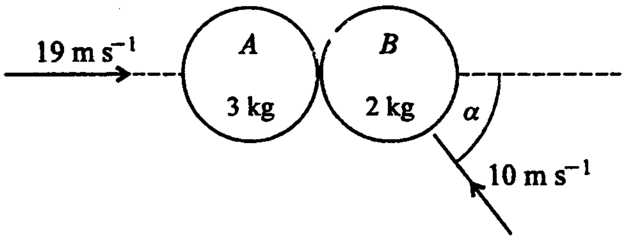
(ii) Find the speed of the particle at the instant when the string becomes slack. [4]

4 A particle $P$ of mass $m$ is attached to a fixed point $O$ by a light inextensible string of length $a$, and is moving in a vertical circle with centre $O$ and radius $a$. Air resistance may be neglected. When $P$ is at the highest point of the circle, the tension in the string is $2mg$.

(i) Find the speed of $P$ when it is at the highest point. [3]

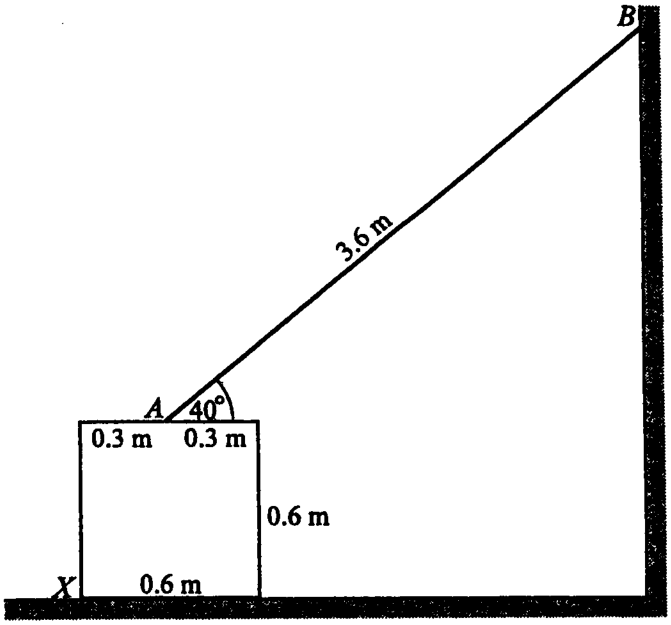
(ii) Find the tension in the string when $OP$ makes an angle of 60° with the downward vertical. [5]

5




Two uniform smooth spheres $A$ and $B$, of equal radius, have masses 3 kg and 2 kg respectively. They are moving on a horizontal surface, and they collide. Immediately before the collision, $A$ is moving along the line of centres with speed $19\,\text{m s}^{-1}$, and $B$ is moving with speed $10\,\text{m s}^{-1}$ at an angle $\alpha$ to the line of centres, where $\sin\alpha = \frac{4}{5}$ (see diagram). The coefficient of restitution between the spheres is 0.4. Find the speed and the direction of motion of each sphere after the collision. [9]

6




A ladder $AB$, of length 3.6 m and mass 15 kg, rests in equilibrium at 40° to the horizontal. Its upper end $B$ is in contact with a smooth vertical wall, and its lower end $A$ is in contact with the rough horizontal top surface of a cubical block. The ladder is modelled as a uniform rod lying in a vertical plane perpendicular to the wall (see diagram).

(i) Find the normal reaction and the frictional force acting on the ladder at $A$, and hence show that the coefficient of friction between the ladder and the block is at least 0.596 (correct to 3 significant figures). [5]

The block has mass $M$ kg and sides of length 0.6 m, and rests on rough horizontal ground. Two of the vertical faces of the block are perpendicular to the wall, and $A$ is in contact with the mid-point of the top surface. The block is on the point of toppling about $X$ (see diagram).

(ii) Find the value of $M$. [4]

(iii) Find the least possible value of the coefficient of friction between the block and the ground. [2]

 [Turn over

7

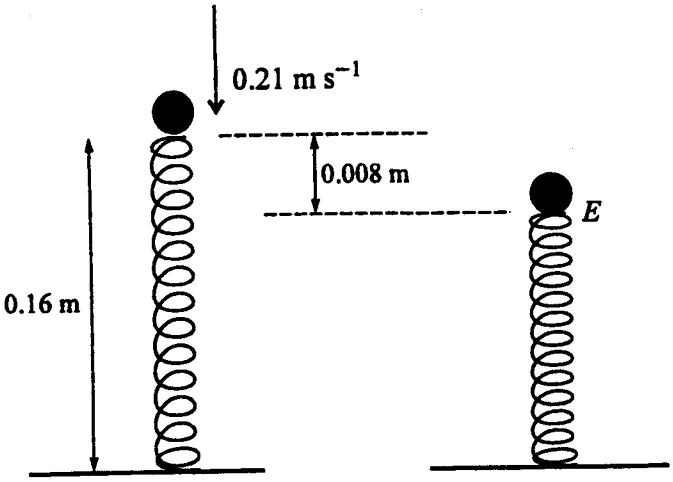



Fig. 1

Fig. 2

A light spring, with natural length 0.16 m and modulus of elasticity 3.92 N, lies in a vertical line with its lower end on the ground. A small stone, of mass 0.02 kg, falls along the same vertical line and when it first makes contact with the spring its speed is 0.21 m s$^{-1}$ (see Fig. 1). The stone continues to move in the vertical line, and remains in contact with the spring until the spring regains its natural length. When the stone is at the point $E$, the spring is compressed by 0.008 m (see Fig. 2). Air resistance may be neglected.

**(i)** Verify that, when the stone is at $E$, its acceleration is zero. [2]

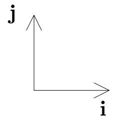
At a time $t$ seconds after the stone first passes through $E$, the stone is $x$ metres below $E$.

**(ii)** Show that, while the stone is still in contact with the spring, $\frac{d^2x}{dt^2} = -1225x$. [3]

**(iii)** Show that the maximum value of $x$ is 0.01. [3]

**(iv)** Find the length of time for which the stone is in contact with the spring. [4]

**1**




**i** component of velocity preserved …

$$\therefore\ 7\cdot 6\cos\theta = 9\cdot 2\cos 40^\circ$$

$$\theta = \mathbf{22\cdot 0^\circ} \qquad \text{(3 s.f.)}$$ [2]

Impulse is in the **j** direction (normal to surface) and …

$$\begin{aligned}|\text{Impulse}| &= 0\cdot 5\,(7\cdot 6\sin 21\cdot 9796...) - 0\cdot 5\,(^-9\cdot 2\sin 40^\circ)\\ &= 4\cdot 3790...\\ &= \mathbf{4\cdot 38\ Ns} \qquad \text{(3 s.f.)}\end{aligned}$$

[3]

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**2**

$$\begin{aligned}m\ddot{x} &= {}^-k\,x^3\\ v\frac{\mathrm{d}v}{\mathrm{d}x} &= {}^-\tfrac{k}{m}\,x^3\\ \int v\,\mathrm{d}v &= {}^-\tfrac{k}{m}\int x^3\,\mathrm{d}x\\ \tfrac{1}{2}v^2 &= {}^-\tfrac{k}{4m}\,x^4 + A\end{aligned}$$

$x = a$, $v = 0$ gives $\quad A = {}^-\frac{ka^4}{4m}$

so speed $= \sqrt{\frac{k}{2m}\left(a^4 - x^4\right)}$

[5]

when $P$ reaches $O$ $(x = 0)$ $\qquad$ speed $= a^2\sqrt{\frac{k}{2m}}$

[2]

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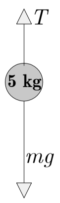
**3**




at the release …

N2($\uparrow$) $\qquad \frac{792}{3\cdot 2}\times 0\cdot 8 - 5\times 9\cdot 8 = 5a$

$$a = 29\cdot 8$$

so the initial acceleration is **29.8 ms$^{-2}$** (upwards) [4]

when the string becomes slack, by conservation of energy …

gain in K.E. = loss in E.P.E. - gain in G.P.E.

$$\begin{aligned}\tfrac{1}{2}\times 5\times v^2 &= \tfrac{1}{2}\times\tfrac{792}{3\cdot 2}\times 0\cdot 8^2 - 5\times 9\cdot 8\times 0\cdot 8\\ v^2 &= 16\end{aligned}$$

**speed = 4 ms$^{-1}$** [4]

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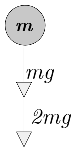
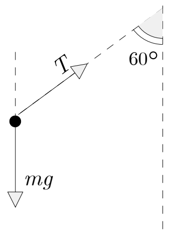
**4**




N2(↓) $\quad mg + 2mg = {mv^2}/{a} \quad\quad v^2 = 3ga$

**speed** $= \boldsymbol{\sqrt{3ga}}$

[3]




energy ....

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}m(3ga) + mg\left(\tfrac{3}{2}a\right)$$

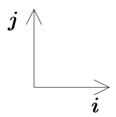
$$v^s = 6ga$$

N2(radially) $\quad T - mg\cos 60^\circ = {m(6ga)}/{a}$

$$T = \boldsymbol{\tfrac{13}{2}mg}$$

[5]

**5**




after the collision ... $\quad \mathbf{v}_A = \begin{bmatrix} u \\ 0 \end{bmatrix} \quad\quad \mathbf{v}_B = \begin{bmatrix} v \\ 8 \end{bmatrix}$

momentum $\quad 3u + 2v = 45$

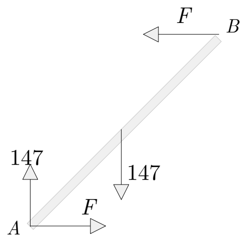
restitution $\quad -u + v = 10$

$u = 5 \quad v = 15$

**velocity of $\boldsymbol{A}$ = 5 ms$^{-1}$ to the right, velocity of $\boldsymbol{B}$ = 17 ms$^{-1}$ at 28·1° above line of centres**

[9]

**6**



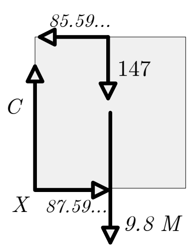

M($B$) $\quad F(3\cdot 6\sin 40^\circ) + 147(1\cdot 8\sin 50^\circ) - 147(3\cdot 6\sin 50^\circ) = 0$

$$F = 87\cdot 5938...$$

$F \le \mu R \quad \Rightarrow \quad \mu \ge {87\cdot 5938...}/{147} = 0\cdot 59587...$

$\boldsymbol{\mu \ge 0\cdot 596}$ $\quad$ 3 s.f. (show)

[5]




M($X$) $\quad 87\cdot 5938...\times 0\cdot 6 \; - \; 147\times 0\cdot 3 \; - \; 9\cdot 8M\times 0\cdot 3 = 0$

$M = 2\cdot 8762... = \mathbf{2\cdot 88}$ $\quad$ (3 s.f.)

[4]

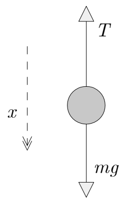
$C = 147 + 9\cdot 8M = 175\cdot 18....$

so for block/ground interaction ....

$\mu' \ge {87\cdot 5938...}/{175\cdot 18...} = \mathbf{0\cdot 500}$ $\quad$ (3 s.f.)

[2]

**7**




N2(↓) $\quad 0\cdot02\,\ddot{x} = 0\cdot196 - \frac{3\cdot92}{0\cdot16}\times 0\cdot008 = 0$

i.e. **acceleration is zero at *E*** [2]

When the stone is $x$ below $E$ (and still in contact with the spring) ….

N2(↓) $$0\cdot02\,\ddot{x} = 0\cdot196 - \tfrac{3\cdot92}{0\cdot16}\times(0\cdot008 + x)$$
$$\ddot{x} = 9\cdot8 - (9\cdot8 + 1225x)$$
$$\ddot{x} = {}^{-}1225x$$
(show) [3]

Hence we have SHM and a solution in the form

$x = a\sin(35t)$ where $t$ is the time after first passing $E$.

Using $v^2 = \omega^2\left(a^2 - x^2\right)$ yields …

$$0\cdot21^2 = 35^2\left(a^2 - \left({}^{-}0\cdot008\right)^2\right) \quad\Rightarrow\quad \boldsymbol{a = 0\cdot01}$$ (3 s.f.)

[3]

Loses contact with the spring when

$$x = {}^{-}0\cdot008$$
$$0\cdot01\sin(35t) = {}^{-}0\cdot008$$
$$\sin(35t) = {}^{-}0\cdot8$$
$$t = 0\cdot063265...$$

so **the stone remains in contact with the spring for 0.0633 s** (3 s.f.)

[4]